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HEATING KINETICS OF A PLANE-PARALLEL LIGHT-SCATTERING LAYER

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A layer heated from both sides by plates as a result of radiative energy transport and a heat-conduction mechanism is considered. A system of differential equations describing radiation transport and heat transfer in the layer is formulated. Its solution is given on the basis of a fixed finite-difference approximation of the spatial derivatives that reduces the problem to a system of ordinary differential equations in time for the temperature and the radiation fluence in the process of heating of both the entire layer and separate portions of it.

The overwhelming majority of artificial and natural objects are more or less light-scattering. Therefore, thermal processes should be investigated with regard for this feature. Depite a number of publications devoted to this problem, only a few satisfactory results allowing one to obtain solutions by comparatively simple technical means have been obtained. Only the development of computer technology has made it possible in recent years to provide rapid quantitative solution of complicated technical problems. Earlier [1], based on differential equations of heat conduction and radiation transport, we analyzed the onset time of the stationary regime in heating of a plane-parallel light-scattering layer as a function of the initial temperatures of the plates, the grids, and the layer, the number of grids, and the optical and heat-conductivity parameters of the light-scattering medium. In what follows, we will analyze the heating kinetics of both the entire layer and separate portions of it.

Let us consider the formulation of the problem. Let there be a horizontally oriented plane-parallel layer bounded on both sides by heated plates. Within the layer, horizontal grids heated to a certain temperature are situated to provide more homogeneous or, conversely, inhomogeneous heating of the layer over its thickness. The thermal regime in the medium is created as a result of both absorption of radiation from the plates and the grids and a heat-conduction process.

Let us define the properties of the medium, the plates, and the grids. From the viewpoint of radiation transport in the layer, it is characterized by the thickness x_0, absorption coefficient k, scattering coefficient s, and scattering indicatrix $\chi(\gamma)$ of an elementary volume of the medium. The parameters k, s, and $\chi(\gamma)$ are phenomenological. They provide information on the properties of the components comprising an elementary volume of the medium. Methods of calculation or measurement of the above parameters can be found in [2-4]. Since we consider a heat-conducting medium, it is also characterized by the elementary-volume-averaged thermal conductivity η, specific heat c, and density ρ. The plates have the temperature T_p, coefficients of reflection r_p and transmission τ_p, and coefficient of heat exchange with the medium h_p. L grids are arbitrarily distributed over the layer thickness. They have the temperature T_g, coefficients of reflection r_g and transmission τ_g, and coefficient of heat exchange h_g. Heat is transferred into the layer as a result of both absorption of radiation from the heated plates and grids and a heat-conduction mechanism.

Analysis of the time-dependent temperature fields is reduced to numerical solution of a system of differential equations of the radiation transport and heat conduction. By using a fixed spatial finite-difference approximation, the partial differential equations are reduced to a system of ordinary differential equations in time for the temperature and the radiation fluences at selected points of the layer considered [5]. Presently, the problem can be solved on a personal computer in the most general formulation, in which each plate and grid have different

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properties, parameters of the heat conduction depend on the time, temperature, and coordinate x, and the optical parameters are also wavelength-dependent. However, a solution of this type will be of doubtful value, since the final results will not allow a simple analysis. The above-formulated problem was solved under the following simplifying assumptions. The temperature of the plates is the same and does not change with time. No grids are present in the layer. Initially, the layer has the same temperature at all points. The optical and thermophysical parameters of the medium are the same at all points and are temperature-independent. The layer and the plates are considered to be black bodies.

We will now formulate the original differential equations and the boundary and initial conditions. Radiation transport in the opaque medium is described by a transport equation. Since in the given problem integral rather than angle-dependent parameters of the radiation are required for analysis of heating of the medium, we use the two-flow approximation of transport theory, which provides a good description of the distribution of irradiance within a plane-parallel layer [6-8]. If the radiation field in the medium is considered to be diffuse, then, according to [7], the following differential equations can be written for the spectrum-integrated downward and upward radiation fluences E_1 and E_2 within the horizontal plane-parallel layer considered with regard for the thermal radiation of the medium:

$$\frac{dE_1}{kdx} = 2(\sigma T^4 - E_1) + \frac{2\Lambda\varphi}{1-\Lambda}(E_2 - E_1), \quad \frac{dE_2}{kdx} = -2(\sigma T^4 - E_2) - \frac{2\Lambda\varphi}{1-\Lambda}(E_1 - E_2). \tag{1}$$

Here $\sigma = 5.67 \cdot 10^{-8}$ W/(m^2·deg^4) is the Stefan–Boltzmann constant, $\Lambda = s/(k + s)$ is the survival probability of a quantum, $\varphi = (3 - x_1)/8$ is the fraction of the radiation backscattered by an elementary volume upon its irradiation from one side by diffuse radiation, and x_1 is the first coefficient in the expansion of the indicatrix $\chi(\gamma)$ in Legendre polynomials.

The heat-conduction equation is as follows:

$$\frac{\partial T}{\partial t} = \frac{k}{c\rho}(E_1 + E_2 - 2\sigma T^4) + \frac{\eta}{c\rho}\frac{\partial^2 T}{\partial x^2}. \tag{2}$$

The boundary conditions for the temperature of the medium at the plates are as follows:

$$\eta \left.\frac{\partial T^{+,-}}{\partial x}\right|_{\mathrm{p}} = \pm h_{\mathrm{p}}(T^{+,-} - T_{\mathrm{p}}), \tag{3}$$

where the plus and minus signs correspond to the temperature in the medium under the upper and above the lower plate. The boundary conditions for the radiation fluences at the upper and lower plates are as follows:

$$E_1^+|_{\mathrm{p1}} = (1 - r_{\mathrm{p1}} - \tau_{\mathrm{p1}})\sigma T_{\mathrm{p1}}^4 + r_{\mathrm{p1}} E_2^+ \tag{4}$$

and

$$E_2^-|_{\mathrm{p2}} = (1 - r_{\mathrm{p2}} - \tau_{\mathrm{p2}})\sigma T_{\mathrm{p2}}^4 + r_{\mathrm{p2}} E_1^- . \tag{5}$$

The initial condition is the constant temperature T_0 throughout the entire medium at the initial instant of time $t = 0$, i.e.,

$$T(0, x) = T_0 . \tag{6}$$

Numerical solution of system (1)-(6) makes it possible to analyze the process of time-dependent transformation of the radiation and temperature fields. The entire space is divided into N zones with the boundaries of the layer lying at grid nodes. Spatial derivatives are replaced by their finite-difference approximations [9]. Here the heat-conduction equation is reduced to a system of N ordinary differential equations, and the system describing

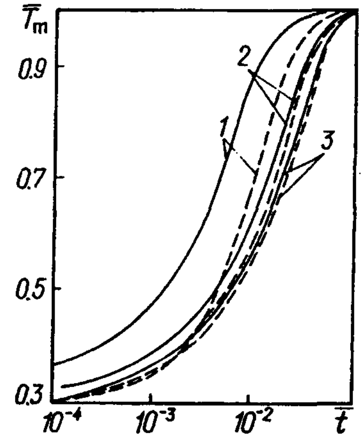

Fig. 1. $\overline{T}_m$ as a function of $\bar{t}$ for different η and h_p/η ($h_p/\eta = 10^5$ m^{-1} (solid curves), $h_p/\eta = 0$ (dashed curves)): 1) η (W·m^{-1}· K^{-1}) = 11.4, 2) 1.14, 3) 0.114.

the radiation transport transforms into a system of $2N$ linear equations. They can be solved by standard procedures. The accuracy of the solution with respect to the coordinate was evaluated for certain variants by doubling the number of nodes. The algorithm described can easily be implemented and does not have high requirements with respect to computational resources. For N of the order of 100, a personal computer is quite suitable for the numerical computations.

We now dwell on results of computations. The process of the temperature variation depends on $\bar{t} = t/c\rho$, and the temperature of the medium can be conveniently normalized by the plate temperature, i.e., it is convenient to consider the relative temperature $\overline{T}_m = T/T_p$. The relationship between the average relative temperature of the layer $\overline{T}_m = (1/x_0)\int_0^{x_0} \overline{T}(x)dx$ and $\bar{t}$ (with the dimensionality W^{-1}·m^{-3}·K) is presented in Fig. 1 on a semilogarithmic scale. It should be noted that the quantity $c\rho$ is of the order of 10^6 J·m^{-3}·K^{-1} for a wide range of media. Therefore, transformation to real time can be carried out by multiplying $\bar{t}$ by 10^6. Here and in what follows the coefficients of reflection and transmission of the plates equal zero, $T_p = 1000$ K, and $T_0 = 300$ K. We consider a disperse layer having the following optical and geometric parameters: $\Lambda = 0.8$, $\varphi = 0.05$, $x_0 = 1$ m, and the absorptive optical thickness $k_{x_0} = 10$. The thermal conductivity had three different values. We refer to media with $\eta = 11.42$, 1.142, and 0.1142 W·m^{-1}·K^{-1} as a conductor, semiconductor, and dielectric, respectively. It should be noted that at high values of η, the heating process depends strongly on the heat-transfer coefficient of the plate–medium system, whereas at low η its value virtually does not affect the average temperature. In the conductor, when h_p is large, ideal conditions for feeding the material with thermal energy are provided as a result of efficient heat transfer from the plate to the medium and its subsequent rapid redistribution over the entire layer thickness. A rapid heating process takes place. If $h_p = 0$, heating takes place only as a result of radiative energy transport. In this case the temperature of the layer increases more slowly. In the dielectric, a change in h_p/η from zero to 10^5 m^{-1} leads to the fact that only the near-surface portion of the medium immediately acquires the plate temperature, which cannot affect strongly the rate of heat propagation into the layer in the case of low thermal conductivity.

We will now consider heating of different portions of the layer characterized by the coordinate $\bar{x} = x/x_0$. Since the two plates have identical properties, the problem is symmetric with respect to the middle of the layer $\bar{x} = 0.5$, and therefore it is sufficient to analyze the temperature kinetics only at points $\bar{x} \leq 0.5$. Dependences of $\overline{T}$ on $\bar{t}$ for four different distances from the plate are presented in Fig. 2 for the dielectric and the conductor. In the same figure, for comparison we present the functions $\overline{T}_m(\bar{t})$ (curves 5). In the dielectric (Fig. 2a), the value of the heat-transfer coefficient does not affect the heating process at virtually all depths. The reason for this was pointed out in describing Fig. 1. Only the layer boundary immediately acquires the temperature of the plate when $h_p/\eta = 10^5$ m^{-1}, whereas when no heat transfer takes place, the heating process slows down. Although the heating kinetics

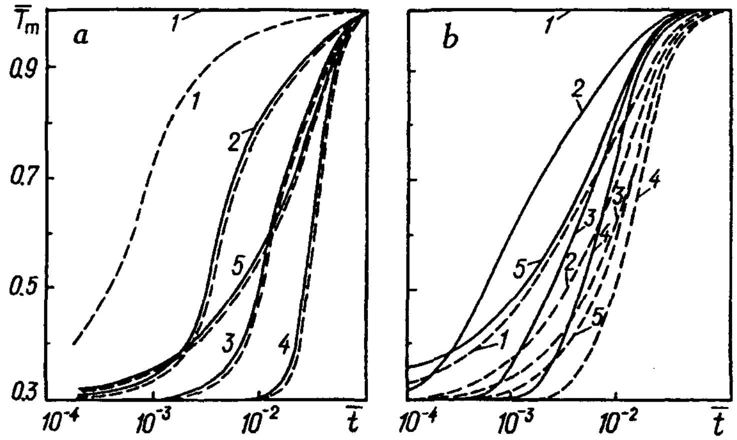

Fig. 2. $\overline{T}_m$ as a function of $\overline{t}$ at different depths in the layer for a dielectric (a) and a conductor (b) ($h_p/\eta = 10^5$ m^{-1} (solid curves), $h_p/\eta = 0$ (dashed curves)): 1) $x = 0$, 2) 0.125, 3) 0.25, 4) 0.5; 5) layer-averaged temperature $\overline{T}_m$.

differs at different depths, and deep-lying layers are, quite naturally, heated more slowly than surface ones, the stationary regime begins everywhere at virtually one and the same time. The temperature change in the conductor has a somewhat different character (Fig. 2b). The effect of the quantity h_p manifests itself in heating of different portions of the layer. Near-surface layers are heated more slowly at first than in the dielectric, since, due to the high thermal conductivity, they transfer heat into the depth. Therefore, deep-lying layers are heated faster. The heat-conduction mechanism leads to more homogeneous and faster heating of the entire layer. However, the scatter in the instants of time of practical onset of the stationary regime at different depths is larger for the conductor than for the dielectric. It should be noted that the character of the change in the average temperature (in both the dielectric and the conductor) is basically different than that at any depth. In the present work we considered extreme cases of the thermal conductivity of the medium. An analysis of intermediate situations would greatly expand the size of the article. However, it is evident that the character of the corresponding regularities will be intermediate between those considered in the present work.

NOTATION

x, current coordinate; $\overline{x}$, normalized layer thickness; x_1, first coefficient of the expansion of the scattering indicatrix in Legendre polynomials; r, reflection coefficient; T, temperature; $\overline{T}$, normalized temperature; h, heat-exchange coefficient of a plate; E_1 and E_2, irradiances from above and from below, respectively; $\overline{t}$, normalized time. Subscripts: p, plate; g, grid; m, layer-averaged.

REFERENCES

1. É. I. Vitkin and A. P. Ivanov, Teplofiz. Vys. Temp., 36, No. 4 (1998).
2. A. P. Ivanov, Optics of Scattering Media [in Russian], Minsk (1969).
3. A. P. Ivanov, S. M. Reprintseva, N. V. Fedorovich, and V. L. Dragun, Inzh.-Fiz. Zh., 22, No. 4, 640-643 (1972).
4. A. P. Ivanov, S. M. Reprintseva, and V. L. Dragun, Dokl. Akad. Nauk BSSR, 16, No. 12, 1094-1097 (1972).
5. É. I. Vitkin and A. I. Zolotovskii, I. P. Smyaglikov, and V. D. Shimanovich, Inzh.-Fiz. Zh., 68, No. 5, 19-26 (1995).
6. É. P. Zege, A. P. Ivanov, and I. L. Katsev, Image Transfer through a Scattering Medium [in Russian], Minsk (1985).
7. É. P. Zege, A. P. Ivanov, and I. L. Katsev, Image Transfer through a Scattering Medium, Berlin (1991).
8. É. P. Zege, Two-Flow Approximation in the Theory of Radiation Transport, Preprint of the Institute of Physics of the Academy of Sciences of the BSSR [in Russian], Minsk (1971).
9. D. Shih, Numerical Methods in Heat-Transfer Problems [Russian translation], Moscow (1988).